

Bidding dynamics in multi-unit auctions: empirical evidence from online auctions of certificates of deposit

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Abstract

This study examines online multi-unit, discriminatory, ascending auctions of certificates of deposit. We find evidence suggesting that the most aggressive bids are likely to occur at the beginning and the end of the auctions. The opening of the auction serves an important role in price discovery. In addition, in multi-unit auctions last-minute bidding is a conditional strategy, and is used only when bidding is intense. Furthermore, we provide evidence suggesting that revenues are increasing in the depth of the market, in the concentration of early bids, and in bank participation relative to the size of the principal.

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1. Introduction

While it is widely perceived that the potential market of business-to-business (B2B) auctions over the Internet is vast,¹ theoretical and empirical research in this area is scant due to the dynamic nature of Internet auction protocols and the usual member-only restriction of access to data, respectively. To better understand the market and its dynamics, this study identifies a unique data set on multi-unit, discriminatory, ascending auctions of certificates of deposit (CDs) held by eight local government Treasurer's offices over the Internet. By analyzing the data set that contains all bids and their distributions over time, we address three important questions:

- (1) What is the nature of price discovery in Internet auctions?
- (2) What is the role played by last-minute bidding (sniping) in auctions?
- (3) What is the relationship between revenue and market depth, and what does this imply about market design?

On the first question, the richness of the data allows us to describe the price discovery of the most aggressive bids in detail. Based on McAfee and McMillian's (1987) argument, "the essential feature" of any ascending auction is the ability of bidders to observe and respond to a subset of the current bids.² Because of the rational expectation of active counter-bidding from opponents, a bidder has no incentive to make a bid close to his or her valuation when the auction has a fixed closing time and is still in its early stage.³ One would then expect the price discovery of the highest bids to occur just before the end of the auction. However, we do not find this to be true in our sample auctions. Specifically, we document that during a typical 30-minute auction session the highest bids from each auction concentrate in the first minute and in the last two minutes. This result suggests that the opening of the auction plays a unique and important role in price discovery.

On the second question, the study demonstrates that last-minute bidding, a practice called "sniping," is not an unconditional strategy. Bajari and Hortacsu (2002) and Roth and Ockenfels (2002) find that bids often arrive very late in online single-unit auctions. They suggest several explanations for this strategic behavior:

- (1) bidders snipe to avoid a bidding war,
- (2) sniping can be an equilibrium of implicit collusion because the behavior has the effect of probabilistically generating higher profits for successful bidders,
- (3) bidders snipe to protect their valuable information when the auction has a common value component, and
- (4) sniping gives opponents little or no time to respond.⁴

¹ According to the Census Bureau of the Department of Commerce, B2B accounted for 95% of all e-commerce in 2000. Goldman Sachs projects B2B sales to reach \$4.5 trillion by 2005. Among all forms of e-commerce, Internet auctions are one of the most successful ones (Bajari and Hortacsu, 2002).

² In practice, not all bids are observable.

³ This reluctance for revealing one's valuation is well documented in the online auction literature, e.g., Bajari and Hortacsu (2002) and Roth and Ockenfels (2002).

⁴ Note that the fourth explanation is also based on the rational expectation of active counter-bidding from opponents. This connection ties the first two inquiries of the paper.

While at first sight these explanations suggest that bidders opt to snipe unconditionally in single-unit auctions, we find that the ratios of last-minute bids to total bids range from zero to 20% with a mode of zero in our sample. By partitioning the sample based on the bidding intensity of the auctions, the study further demonstrates that sniping is necessary and desirable in multi-unit auctions only when the bidding is intense.⁵

Third, the study finds evidence suggesting that revenues are increasing in the depth of the market (proxied by the amount of the auctioned principal), in the concentration of early bids, and in bank participation relative to the size of the principal. These results have important implications on market design. For example, it is beneficial to consolidate several small auctions into a larger one because revenue is increasing in the amount of the auctioned principal. In addition, the positive relation between early bidding and revenue supports the use of activity rules that enforce early-bid submission.⁶

Most pre-Internet auction research was either interested in single-unit theories or conducted in laboratory settings. By foregoing spatial and temporal constraints and providing ready access to information, the Internet has shaped the intermediation economy and the auction literature.⁷ A fast growing literature on Internet auctions, e.g., Baye et al. (2002), Bajari and Hortacsu (2002), Biais and Faugeron-Crouzet (2002), Brown and Goolsbee (2002), Hasker et al. (2001), Lucking-Reiley (1999), and Roth and Ockenfels (2002), has utilized the high-quality data available on the Internet and provided new insights into auction dynamics. However, these empirical studies mainly focused on consumer-to-consumer and business-to-consumer auctions in which the selling items are mostly single-unit. Thus, one purpose of the paper is to fill the gap with respect to the lack of multi-unit results.

By analyzing CD auctions, the second purpose of the paper is to highlight the role of the Internet on the evolution of financial intermediaries. According to Grant Street Group, an online fixed-income and equity securities auction firm from which we obtain our auction data, the firm has held more than 4000 auctions and generated approximately three trillion auction dollars since November 1997. In addition, many notable B2B web sites, including eSpeed (wholesale fixed-income securities by Cantor Fitzgerald), Intercontinental Exchange (energy, metal, and other commodities by Goldman Sachs, Morgan Stanley Dean Witter, Deutsche Bank, BPAmoco, Royal Dutch/Shell Group, etc.), Chemconnect (chemicals), e-Steel (steel), Paperspace (paper and pulp), and DRAMeXchange (DRAMs), are actively auctioning and trading spot, forward, and derivative contracts. They are either competing with or complement the so-called “old-economy” financial intermediaries with a new definition of business scope that expands pervasively into a new set of assets, e.g., chemicals, paper, and DRAMs. Their market potential warrants scholar attention and analysis.

⁵ Bidders may care more about how early or late bidding affects their probability of having their bids accepted at the margin. As a result, the timing and distribution of the highest bids may not have much of an influence at the margin on many bidders' payoffs.

⁶ For dynamic auctions, there have been concerns that auction rules should not give bidders an incentive to delay their bid submission. For this reason, the Federal Communications Commission (FCC) spectrum auctions specify that a bidder who does not maintain active status on a certain volume of spectrum licenses would not be able to suddenly become active on a large volume of spectrum licenses late in the multi-unit, simultaneous auctions.

⁷ See Chemmanur and Wilhelm (2002) for a discussion about the impact.

The paper is organized as follows. Section 2 describes the auction rules and the data. Section 3 presents descriptive analysis. Section 4 analyzes the determinants of revenues. The final section concludes the paper.

2. The auction rules and the data

In October 1999, the Ohio Treasurer's office started a "BidOhio" internet auction program, which was powered by Grant Street Group (then called Muniauction.com), to use the State's interim funds to purchase certificates of deposit from Ohio depository institutions. Since then, a total of eight local Treasurer's offices, including Idaho, Iowa, Louisiana, Pennsylvania, South Carolina, Texas, and the Allegheny County of Pennsylvania, have adopted similar programs. As of 07/20/2002, Grant Street Group has hosted a total of 100 CD auctions. Our data set contains the most recent 76 auctions during the time period of 02/06/2001–07/17/2002 because the disclosure on these sample auctions is more complete, allowing for a more intense analysis.

CD auctions fall into a broad category of B2B e-commerce; individual investors do not have direct accesses to these transactions. These CDs are mostly six-month deposits with a minimum accepted rate set to be either the Treasury bill rate or Treasury bill bond equivalent yield of the same maturity. Thus, the minimum rate is virtually the risk-free rate. Among the 76 sample auctions, 23 auctions clear at their minimum rates. Among these 23 auctions, 15 of them have unsold balances. For these "cold" auctions, the average unsold balance is \$14.48 million, and the average unsold rate is 38.48%.

In general, only local depository institutions are eligible to register with the auctioneer. These bidders range from nation-wide, leading commercial banks to community savings banks and trusts. While it is legitimate for a bank with operations in multiple states to participate in auctions held by different Treasurer's offices, a casual examination of the data suggests that most banks do not participate in cross-bidding. In addition to registration, banks also need to request and receive admission to an auction to submit bids. Access is protected by user identification and a password. The auctions are advertised in advance.

Our sample online auctions are mostly held for a 30-minute period.⁸ Upon acceptance to an auction, each bank may submit a schedule of up to five rate–quantity pairs, each with a minimum amount of \$100,000 and an increment of \$100,000. For example, bank A may submit five rate–quantity pairs: 2.00%–\$1,000,000, 2.10%–\$1,000,000, 2.20%–\$1,000,000, 2.30%–\$1,000,000, and 2.40%–\$1,000,000. The maximum amount of funds that can be allocated to a bank is limited to a typical \$5 million cap for political and risk diversification reasons.⁹ Bidders are not allowed to form bidding syndicates. The final allocation of funds follows price–time priority. That is, in order for a bid to be at least partially filled, it must be submitted with a rate that is no less than the current clearing rate. If qualified bids submitted by the same or different bidders result in a tie, the earlier

⁸ In our sample, one auction has a shorter auction period of 15 minutes. Later, when analyzing the data, the study normalizes the time stamps by the lengths of their auction period so that they are on the same scale.

⁹ Allegheny County and the State of Texas have caps of \$4 and \$7 million, respectively. Individual caps can be further reduced if a bidder's credit rating and collateral does not fulfill requirements.

bid prevails. The auction is discriminatory in which bidders pay the rate they offer on their winning bids.

During an auction, bids are made by entering rate–quantity pairs on the auction web page. Bidders do not have information on opponents' individual bids, but bidders can check the status of their own rate–quantity pairs at any time to see whether these pairs are still “in-the-money,” i.e., whether the rates are still accepted. The auctioneer also continuously notifies the bidder of the dollar amounts that are in-the-money for the bidder's schedules. If a pair is “out-of-the-money,” the bidder can choose to improve the pair. There is no limit as to the number of times a rate–quantity pair may be updated. However, once a rate–quantity pair is submitted, the bidder may only increase, but not reduce, the rate and the amount of the pair. For example, if the current clearing rate is 2.05%, bank A's first rate–quantity pair would be out-of-the-money. If bank A decides to improve this pair, the bank may increase its rate to 2.10%. This revision increases the total bids submitted from five to six in which five are active and one is stale. Once the auction is over, the auctioneer posts all *final* bids with bid amounts, bidder identifications, and time stamps. Summary statistics, including the number of banks bidding, the total number and dollar amount of bids received, and the value-weighted accepted rate, are tabulated and posted. In addition, a bid graph that plots *all* bids, not just final bids, and the evolution of the lowest rate accepted and the value-weighted accepted rate against the typical 30-minute auction period is also available for the more recent sample auctions.

Table 1 presents summary statistics of the data. The principal has a mean of \$40.45 million and a standard deviation of \$24.59 million. This deviation is largely due to the deviation between Treasurer's offices because it is often the case that all auctions held by the same Treasurer's office are for the same dollar amount. The dollar amount of bids received has a mean of \$62.21 million and a standard deviation of \$46.96 million. Because the standard deviation of the dollar amount of bids received is proportionally larger than the standard deviation of the principal, it is evident that the demand for funds across auctions is not proportional, i.e., we have both “cold” and “hot” auctions with different degrees of bidding intensity in our sample. During the sample period, the minimum rate and the

Table 1
Summary statistics

	Mean	Standard dev.	Skewness	Kurtosis
Principal (million)	40.45	24.59	−0.03	−1.42
\$ of bids received (million)	62.21	46.96	0.68	−0.72
Minimum rate (%)	2.67	1.10	0.78	−0.85
Lowest rate accepted (%)	2.75	1.04	0.83	−0.71
Weighted accepted rate (%)	2.80	1.06	0.83	−0.70
# of banks bidding	21.61	15.98	0.69	−1.08
# of banks accepted	17.00	12.70	0.80	−0.67
# of bids received	140.51	171.10	1.71	2.96
# of early bids	17.93	17.56	1.17	0.33
# of late bids	8.43	8.86	1.21	0.77

This table presents the summary statistics of the data. The data contains 76 auctions of certificates of deposit during the time period of 02/06/2001–07/17/2002. Bids that are submitted in the first five minutes and the last minute are considered as early bids and late bids, respectively.

lowest accepted rate is on average 2.67% and 2.75% with a standard deviation of 1.10% and 1.04%, respectively. Loosely, the use of Internet auctions helps Treasurer's offices improve their revenues by 13 basis points, as measured by the difference between the value-weighted accepted rate and the minimum rate.¹⁰

On average, there are 21.61 banks bidding in an auction while 17.00 banks are accepted. Based on these numbers, it appears that bidding for funds is not particularly intensive for at least a subset of the sample auctions. The average number of bids received for an auction is 140.51, indicating that each bank submits an average of 6.50 bids in an auction. However, note that the standard deviation of the number of bids received is skewed to the right and has a large value of 171.10. This suggests that in many auctions each bidder on average enters slightly less than 6.50 bids, while in some "hot" auctions each bidder on average submits many more bids.

On average, an auction yields 17.93 bids and 8.43 bids in the first five minutes and in the last minute of the 30-minute auction period, respectively. Normalized by the total number of bids received, the bidding concentrations are 12.76% and 6.00% for the first five minutes and the last minute, respectively. Because in an auction where bids are evenly distributed over a 30-minute time period, one would expect the bidding concentrations to be 16.67% and 3.33% for the first five minutes and the last minute, respectively. The evidence suggests that early-bid submission tends to be light and late-bid submission tends to be intense.

3. Descriptive analysis

Two distinct features of the CD auction market make the following descriptive analysis particularly economically meaningful. First, these bidders are professional and knowledgeable in auctioned goods. Thus, the results are likely to be representative for other B2B multi-unit auctions in which only professional, corporate bidders participate. Second, since only registered bidders are allowed to submit bids, the common problem of shilling, a strategy of placing phony bids by the (syndicate of) sellers to run up the closing price as often seen on eBay and Amazon, is free from our data set.

3.1. The most aggressive bids

The most aggressive bids are an interesting subset of bids. Compared with the other bids, they have the largest contribution to revenue maximization. As a result, by studying the price discovery of highest bids, we are more likely to extract the price discovery of revenues.

Figure 1 is the histogram of highest bids; these bids are normalized by subtracting the minimum rates from them. The results based on the full sample of 76 auctions are presented in the top panel. It is evident that the highest bids are skewed to the left. Most of the highest bids have a value less than 0.4%. However, six highest bids are 1% above their minimum

¹⁰ According to Joseph T. Deters, Ohio Treasurer of State, "before interim Treasury funds are auctioned over the Internet . . . banks would place minimum bids with the knowledge that, regardless of their offer, each would receive part of the month's deposits . . . the yield was lower than what would be achieved in a competitive market."

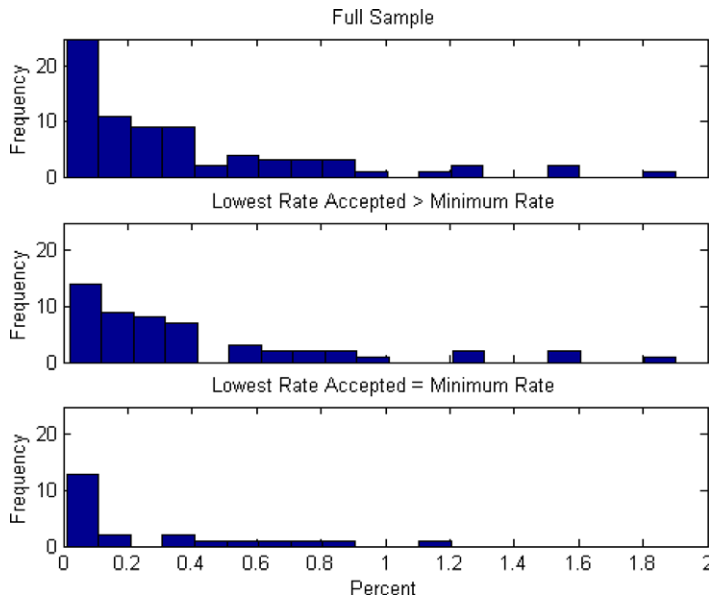


Fig. 1. The histogram of highest bids (net of the minimum rates).

rates. Because the mean of differences between the minimum rates and the lowest accepted rates is merely 0.08%, these aggressive bidders appear to pay too much and fail to observe and respond to the bidding information available to them. In the middle panel, we plot the histogram for the subset of “hot” auctions in which the lowest rate accepted is higher than the minimum rate. The figure shows that the subset has a higher center than the full sample. This implies that for the remaining “cold” auctions, as showed in the bottom panel, the average value of their normalized highest bids is relatively closer to zero. However, despite being cold, there are 5 out of the 23 cold auctions that have a normalized highest bid of greater than 0.5%. Again, comparing this magnitude to the mean of differences between the minimum rates and the lowest accepted rates, 0.08%, they seem quite high.

Figure 2 plots the distribution of the highest bids over the typical 30-minute auction period. The top panel shows that the highest bids exhibit a U-shaped pattern in which they concentrate in the first minute and the last two minutes of the auctions. In addition, the mode of the distribution is located at the interval of 0–1 minute, suggesting that the seeming overbidding is frequently made at the very beginning of the auctions. The result is in contrast to the rational notion that bidders have no incentives to make bids close to their valuations when the auction has a fixed closing time and is still in its early stage (Bajari and Hortacsu, 2002; Roth and Ockenfels, 2002). The evidence also suggests that the opening of the auction plays a unique role in the price discovery of valuations.

The result is robust even we use different cutoffs to define what constitute early bids and late bids. Specifically, there are 23 and 5 highest bids occurring in the first five minutes and the last minute, resulting in the highest bidding concentrations of 30.26% and 6.58% for the first five minutes and the last minute, respectively. Comparing them to the bidding con-

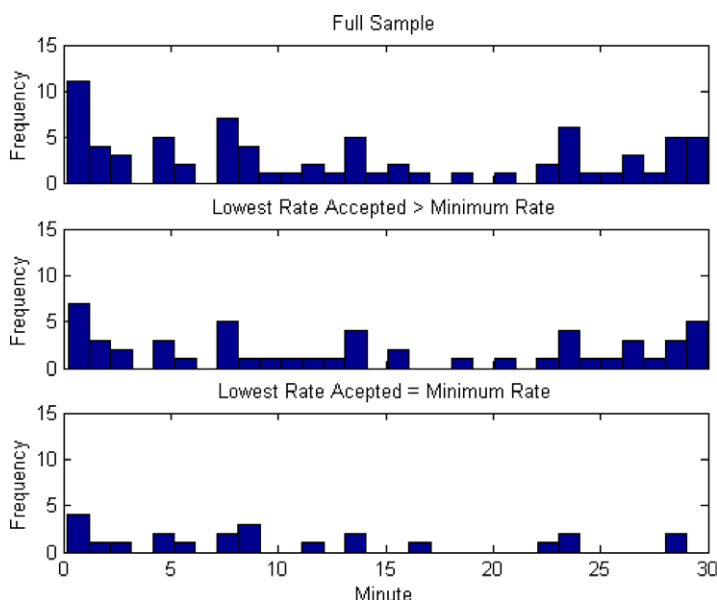


Fig. 2. The distributions of highest bids.

centrations of 12.76% and 6.00% for the first five minutes and the last minute, respectively, it is apparent that early bids tend to be the highest bids.

The fact that the highest bids frequently come early in the auction is quite different from what is observed in other dynamic, ascending auctions. One possible explanation for this is that these early highest bids concentrate in cold auctions due to the lack of later bids in these auctions. Another possibility is that there is collusion in these auctions, and bidders desire to sell large quantities of CDs to secure a low price. While it is difficult to distinguish between these explanations because testing for collusion in multi-unit auctions is a complicated issue,¹¹ we partition the full sample into two sub-samples according to whether the lowest accepted rate is greater than the minimum rate. The idea is that if early highest bids are due to the lack of later bids in some auctions or to collusion, one would expect the subset of auctions with the lowest accepted rate equal to the minimum rate to be characterized by a higher degree of early highest bids. However, we do not observe this. The distributions for the two sub-samples are plotted in the bottom two panels of Fig. 2. Both distributions spread over the whole domain.

A question arises. If not for the lack of bidding participation or for collusion, why bid so high at the beginning of the auctions while more information may be revealed in the latter part of the auctions and can be incorporated into bidding strategies? To shed some light, for each Treasurer's office we track the identifications of the highest bidders from the current auction to the next auction and analyze the frequency by which the most aggressive bidder in the current auction is also the most aggressive bidder in the next auction. We find

¹¹ Testing for collusion is thought to be a difficult question even in single-unit auctions (Bajari and Ye, 2003).

weak evidence of continuous overbidding. Specifically, among the 68 consecutive pairs of auctions held by the same Treasurer's office, there are 11 repeated highest bidders. This results in a realized frequency of 16.18%. In contrast, because there is an average of 12.11 bidders in the latter auctions of the 68 pairs of auctions, the upper bound of the probability that a bidder may *randomly* become the highest bidder is 8.25%, which is only about half of the realized frequency. The reason that this measure is an upper bound is that some previous highest bidders may choose not to participate in the latter auctions. Due to this overestimation and the fact that realized frequency is not normally distributed, the testing results of the *t*-test of paired comparisons for two population means are likely to be weak and can only be regarded as approximate. The value of the *t*-statistic is 1.64 and it is statistically significant at the 10% level, suggesting that there is a tendency for the previous highest bidder to submit the highest bid in the following auction.

The tendency for continuous overbidding and for highest bids to frequently come early in the auction is quite puzzling. While providing an answer is beyond the scope of the paper, we can immediately exclude the following possibilities. First, CD bidders are depository institutions who are presumably highly professional and are unlikely to make repeated mistakes. Second, most of these high bids cannot solely be due to human errors during bid entering. For each bid submitted, a separate verification page will automatically appear and the bidder must verify it. Furthermore, as the *t*-test suggests, these aggressive bids are not random and, therefore, are unlikely due to entering errors. Third, bidder asymmetry in costs and demand elasticity does not seem to be able to fully explain the puzzle. The reason for this is that although the documented asymmetry in [Bajari and Ye \(2003\)](#) and [Hortacsu \(2002a, 2002b\)](#) is useful in explaining the relationship between bidders' private information and their bids in sealed-bid auctions, our sample auctions are ascending.¹² So long as bidders rationally expect active counter-bidding from opponents, they should have no incentive to make a bid close to their valuations with respect to their costs and demand elasticity when the auction has a fixed closing time and is still in its early stage. Fourth, although it is always possible some bidders may intentionally bid high, the reputation for being aggressive should yield little benefits. Unlike in take-all-or-nothing single-unit auctions, it is inconceivable that aggressive bidding can effectively discourage the participation of opponents in a multi-unit setting with strict allocation caps.

3.2. Sniping?

The study operationally quantifies the last-minute bidding as the ratio of the number of bids submitted in the last minute to the number of total bids. [Figure 3](#) plots the distributions of last-minute bids. For an auction with evenly distributed bids over a 30-minute bidding period, we would expect these ratios to have a center of 3.33%. In the top panel, the histogram of the sniping ratios for the full sample clearly indicates that the distribution does not center around 3.33%. Instead, it consists of a spike of zero and a seemingly normal distribution with a mean of about 9%. Its range is from zero to 20%. The mode of

¹² [Dasgupta and Tsui \(2003\)](#) show that matching auctions may result in higher revenues than the standard auctions in a common-value setting because of differences in initial toeholds and asymmetric effects of the bidders' private signals on value.

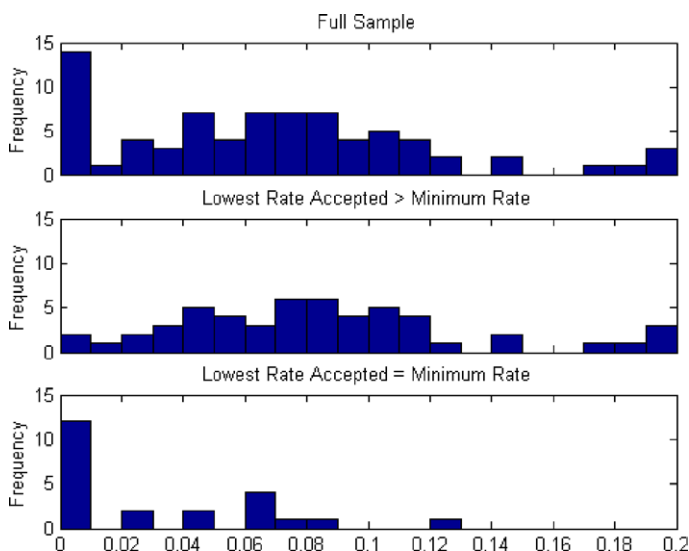


Fig. 3. The distributions of last-minute bids. The values plotted in these histograms are the ratios of the number of last-minute bids to the number of total bids.

the distribution is located at zero, indicating that a large number of auctions end with no last-minute bids. Thus, consistent with [Bajari and Hortacsu \(2002\)](#) and [Roth and Ockenfels \(2002\)](#), bids do not distribute evenly over time and last-minute bids appear to contain a strategic component. However, the spike at zero suggests that, unlike the single-unit auctions examined by [Bajari and Hortacsu \(2002\)](#) and [Roth and Ockenfels \(2002\)](#), multi-unit auctions seem to display a richer behavioral content of sniping.

To check this, we partition the full sample into two sub-samples according to whether the lowest accepted rate is greater than the minimum rate. A unique feature in our sample is that the minimum rate is set at either the Treasury bill rate or the Treasury bill bond equivalent yield of the same maturity. Since in equilibrium only the US Treasury can borrow at the risk-free rate due to default risk, the lowest accepted rate in an auction serves as a narrow, but clean measure of bidding intensity. The bottom two panels of [Fig. 3](#) show that sniping is conditional upon whether bidding is intense. When bidding is rather intense, the distribution of last-minute bids is quite close to a normal distribution with a center around 9%. This concentration is approximately two times that of under an even distribution. However, when the bidding is not intense, the distribution is characterized by the spike at zero. Most of these auctions end with few or no last-minute bids.

In short, in multi-unit auctions sniping is a conditional strategy. Sniping is necessary only when the uncertainty of not acquiring auctioned goods is high.

4. The determinants of revenues

This section investigates the determinants of revenues. Since Treasurer's offices should earn a rate that is no less than the risk-free rate, the difference between the value-weighted

accepted rate and the minimum rate serves as a first-order approximation of revenue. The study chooses three *ex ante* characteristics as potential determinants of revenues: the natural logarithm of the principal, the ratio of the number of banks bidding to the principal, and the ratio of the number of early bids to the principal.

Holding other factors constant, an increase in the supply of the principal should result in a lower rate and the same rate under the traditional bidding models and the traditional financial theories, respectively. However, the microstructure literature suggests that when stand-by limited orders provide good market depth, i.e., more supply and more demand, the market is more liquid, which in turns improves asset prices. Therefore, the expected sign for the natural algorithm of the principal is mixed.

The ratio of the number of bank bidding to the principal is a measure of bank participation.¹³ The importance of bidder participation has long been recognized in the auction literature. It is frequently suggested that bid shading is related to the number of active bidders. Since each bidder is subject to an allocation cap in our sample, we normalize the number of bank bidding by dividing it with the principal. The expected sign for the ratio is negative.

Activity rules have been adopted in a variety of auctions. The most famous ones are those used in the FCC spectrum auctions. While the idea of using activity rules to facilitate price discovery is appealing, no empirical evidence has been provided to support the use of these rules. The study examines the relationship between revenue and the ratio of the number of first five-minute bids to the principal. If early participation is important, we would expect that the higher the ratio, the higher the revenue. That is, a positive sign for the ratio is anticipated.

The univariate ordinary least squares (OLS) regression results are presented in panel A of Table 2.¹⁴ The three variables are all statistically significant at the 1% level. The adjusted R-squared values range from 5.67% to 14.93%. The natural logarithm of the principal has a positive sign, indicating that there is an economy of scale in the supply of funds. The result is consistent with the prediction of the microstructure literature that an increase in the supply of funds along a symmetrical increase in the demand of funds improves liquidity. An implication of the result is that it may be beneficial to consolidate several small auctions into appropriately sized ones. As expected, the signs for the ratio of the number of banks bidding to the principal and the ratio of the number of early bids to the principal are positive. The results show that bank participation and early bidding improve revenues. The evidence indirectly supports the use of activity rules.

In panel A of Table 2, we also report multivariate OLS regression results. Because the correlation coefficient between the ratio of the number of banks bidding to the principal and the ratio of the number of early bids to the principal is 0.7739, we focus on two bivariate regressions that exclude either one of the two variables. The regression results confirm the robustness of explanatory ability of these determinants. They maintain their signs and improve their statistical significance. Although it is problematic to run a regression that

¹³ Loosely, this ratio can also be used to proxy for marketing efforts for attracting bidders. Therefore, the ratio may capture the joint effects of bank participation and marketing efforts.

¹⁴ Admittedly, these regressors are arguably endogenous. As a result, the use of OLS regression has its own limitations.

Table 2

The determinants of revenues

Intercept	log(principal)	# of banks Principal	# of early bids Principal	Adj. R^2 (%)
Panel A: The value-weighted accepted rate minus the minimum rate				
−0.0789 (−1.71)	0.0611 (4.27)**			14.93
0.0619 (3.17)		0.1140 (3.42)**		5.67
0.0756 (3.89)			0.1158 (3.20)**	11.24
−0.4253 (−7.16)	0.1133 (8.29)**	0.2826 (6.95)**		45.49
−0.2334 (−6.55)	0.0834 (7.75)**		0.1673 (6.47)**	38.23
−0.4010 (−6.03)	0.1093 (7.42)**	0.2235 (3.36)**	0.0522 (1.44)	45.74
Panel B: The lowest accepted rate minus the minimum rate				
−0.0378 (−0.98)	0.0358 (2.93)**			6.46
[0.2803]	[0.0493]*			
0.0240 (1.39)		0.1013 (4.57)**		6.41
[0.2187]		[0.0023]**		
0.0376 (2.28)			0.0999 (3.16)**	11.79
[0.6405]			[0.0004]**	
−0.2990 (−5.76)	0.0751 (5.88)**	0.2131 (6.96)**		30.43
[< 0.0001]	[< 0.0001]**	[< 0.0001]**		
−0.1606 (−5.00)	0.0535 (5.50)**		0.1330 (5.17)**	26.83
[0.0005]	[0.0003]**		[< 0.0001]**	
−0.2729 (−4.83)	0.0709 (5.32)**	0.1498 (2.61)**	0.0559 (1.53)	31.08
[< 0.0001]	[< 0.0001]**	[0.0029]**	[0.3048]	

This table presents the regression results of revenues on the characteristics of the auctions. The dependent variable is either the difference between the value-weighted accepted rate and the minimum rate or the difference between the lowest accepted rate and the minimum rate. The independent variables include the natural logarithm of the principal, the minimum rate, the ratio of the number of banks bidding to the principal, and the ratio of the number of early bids to the principal. All *t*-statistics are corrected for heteroskedasticity and contained in parentheses. The *p*-values from tobit regressions are contained in brackets.

* Statistical significance at the 5% level.

** Statistical significance at the 1% level.

includes both the ratio of the number of banks bidding to the principal and the ratio of the number of early bids to the principal, we find that the three determinants maintain their signs.

In panel B of Table 2, we use the difference between the lowest accepted rate and the minimum rate as the dependent variable and repeat the above regression analysis. We do

so because, as shown in [Table 1](#), the lowest accepted rate is close to the value-weighted accepted rate. Therefore, this new measure can be used to check the sensitivity of our baseline regression results. Furthermore, our baseline regression analysis is plagued by the fact that the dependent variable is censored. When the difference between the lowest accepted rate and the minimum rate is used as the dependent variable, one is able to exactly specify zero as the cutoff and use tobit regressions to mitigate the censoring problem. Therefore, we also report tobit regression results with the use of the new dependent variable. The OLS regression results show that the explanatory ability of the logarithm of the principal, the ratio of the number of banks bidding to the principal, and the ratio of the early bids to the principal are largely sustained. The tobit regression results are also mostly consistent with the OLS regression results, indicating that the censoring problem seems to have little impact on statistical inferences.

5. Conclusions

This study examines online multi-unit, discriminatory, ascending auctions of certificates of deposit. We find evidence suggesting that the most aggressive bids are likely to occur at the beginning and the end of the auctions. The opening of the auction plays a unique role in the price discovery of valuations. In addition, our findings indicate that in multi-unit auctions sniping is a conditional strategy. Bidders snipe only when bidding is intense. Moreover, the empirical results suggest that revenues are increasing in the depth of the market, in the concentration of early bids, and in bank participation relative to the size of the principal.

Our results have policy implications. For instance, since the opening of the auction has a unique role in price discovery, an advertisement during the auction should be less effective than the one before the auction. It is possible that an extreme activity rule that automatically grants admission for a bidder to an auction only if he or she enters a bid that is no less than the minimum rate before or at the opening of the auction may facilitate price discovery. The result that revenues seem to be increasing in the concentration of early bids also supports the use of activity rules. Finally, our results suggest that there is an economy of scale in the principal.

Our approach is descriptive and exploratory due to the lack of theories on multi-unit, discriminatory, ascending auctions.¹⁵ Because many online auctions are conducted in the same or a similar auction format, future theoretical development in this area should be fruitful. Another interesting research question unaddressed in this study is the impact of allocation caps on bidding dynamics and revenues. Most of the existing multi-unit theories allow for 100% of auctioned goods to be granted to a single bidder if the bidder is aggressive enough. However, in the real world, either due to political reasons or the diversification consideration of non-performing risk, multi-unit auctions are likely to impose allocation caps. We hope that our work can stimulate future research to address these issues.

¹⁵ A lack of theoretical work in the area of multi-unit auctions have been echoed by [McAfee and McMillan \(1987\)](#), [Milgrom \(1989\)](#), and [Ausubel and Cramton \(2002\)](#).

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